ALBACER
ACADEMIA DE ESTUDIOS

Evaluación para el acceso a la Universidad

Curso 2022/2023 (Julio)

Materia: Matemáticas II

1

$$\left.\begin{array}{l}-2x+y-z=-1\\ -x+ay+z=2\\ 2x+y+az=3\end{array}\right\} \quad M=\begin{pmatrix}-2 & 1 & -1\\ -1 & a & 1\\ 2 & 1 & a\end{pmatrix} \quad M^*=\left(\begin{array}{ccc|c}-2 & 1 & -1 & -1\\ -1 & a & 1 & 2\\ 2 & 1 & a & 3\end{array}\right)$$

a)

$$|M|=\begin{vmatrix}-2 & 1 & -1\\ -1 & a & 1\\ 2 & 1 & a\end{vmatrix} = -2a^2+1+2+2a+2+a = -2a^2+3a+5$$

$$-2a^2+3a+5=0$$

$\boxed{a=\frac{5}{2}}$ $\boxed{a=-1}$

Para $a \notin \{-1, \frac{5}{2}\}$, $Rg\,M = Rg\,M^* = 3 =$ nº incógnitas $\longrightarrow$ S.C.D

$\boxed{a=-1}$

$$|M^*|=\begin{vmatrix}1 & -1 & -1\\ -1 & 1 & 2\\ 1 & -1 & 3\end{vmatrix} = \cancel{3}-\cancel{1}-\cancel{2}+\cancel{1}+2-\cancel{3}=0 \longrightarrow$$

$$|M^*|=\begin{vmatrix}-2 & -1 & -1\\ -1 & 1 & 2\\ 2 & -1 & 3\end{vmatrix} = -6-1-4+2-4-3=-16 \neq 0 \longrightarrow Rg\,M^*=3$$

Para $a=-1 \rightarrow Rg\,M = 2 \neq Rg\,M^* = 3 \rightsquigarrow$ S.I.

$\boxed{a=\frac{5}{2}}$

$$|M^*|=\begin{vmatrix}1 & -1 & -1\\ 5/2 & 1 & 2\\ 1 & 5/2 & 3\end{vmatrix} = 3-\frac{25}{4}-2+1-5+\frac{15}{2} = -\frac{7}{4} \neq 0 \longrightarrow Rg\,M^*=3$$

Para $a=\frac{5}{2} \rightarrow Rg\,M = 2 \neq Rg\,M^* = 3 \rightsquigarrow$ S.I.

b) $M = \begin{pmatrix} -2 & 1 & -1 \\ -1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ $\quad M^* = \left(\begin{array}{ccc|c} -2 & 1 & -1 & -1 \\ -1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 3 \end{array}\right)$

Para $a = 2 \rightarrow$ S.C.D.

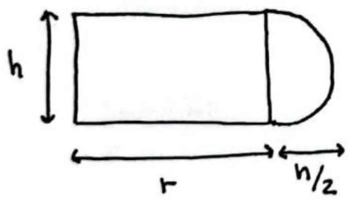
$$|M| = \begin{vmatrix} -2 & 1 & -1 \\ -1 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -8 + 1 + 2 + 4 + 2 + 2 = 3$$

$$x = \frac{\begin{vmatrix} -1 & 1 & -1 \\ 2 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}}{3} = \frac{-4 - 2 + 3 + 6 + 1 - 4}{3} = \boxed{0}$$

$$y = \frac{\begin{vmatrix} -2 & -1 & -1 \\ -1 & 2 & 1 \\ 2 & 3 & 2 \end{vmatrix}}{3} = \frac{-8 + 3 - 2 + 4 + 6 - 2}{3} = \boxed{\frac{1}{3}}$$

$$z = \frac{\begin{vmatrix} -2 & 1 & -1 \\ -1 & 2 & 2 \\ 2 & 1 & 3 \end{vmatrix}}{3} = \frac{-12 + 1 + 4 + 4 + 4 + 3}{3} = \boxed{\frac{4}{3}}$$

2)



a) $P = 2r + h + \frac{\cancel{2}\pi \text{radio}}{\cancel{2}} = 2r + h + \pi \cdot \frac{h}{2}$

$$2r + h + \pi \cdot \frac{h}{2} = 80$$

$$r = \frac{80 - h\left(1 + \frac{\pi}{2}\right)}{2}$$

$$A = h \cdot r + \frac{\pi \cdot \left(\frac{h}{2}\right)^2}{2}$$

$$A = h \cdot r + \frac{\pi \cdot \frac{h^2}{4}}{2}$$

$$A = h \cdot r + \frac{\pi \cdot h^2}{8} \longrightarrow A = h \cdot \left(\frac{80 - h\left(1 + \frac{\pi}{2}\right)}{2}\right) + \frac{\pi \cdot h^2}{8} = \frac{80h}{2} - \frac{h^2\left(1 + \frac{\pi}{2}\right)}{2} + \frac{\pi \cdot h^2}{8} =$$

$$= 40h - \frac{h^2\left(1 + \frac{\pi}{2}\right)}{2} + \frac{\pi h^2}{8}$$

b) $A' = 40 - \frac{1}{\cancel{2}} \cdot \cancel{2} h\left(1+\frac{\pi}{2}\right) + \frac{2\pi}{8} h$

$A' = 40 - h\left(1+\frac{\pi}{2}\right) + \frac{\pi}{4} h$

$40 - h\left(1+\frac{\pi}{2}\right) + \frac{\pi}{4} h = 0$

$-h\left(1+\frac{\pi}{2}\right) + \left(\frac{\pi}{4}\right) h = -40$

$-1{,}78 h = -40$

$h = \frac{-40}{-1{,}78} = \boxed{22{,}4}$

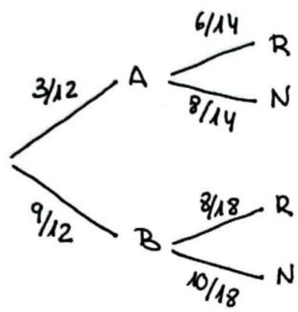
$r = \frac{80 - h\left(1+\frac{\pi}{2}\right)}{2}$

$r = \frac{80 - 22{,}4\left(1+\frac{\pi}{2}\right)}{2}$

$\boxed{r = 11{,}2}$

3.

a)



a.1)

$P(R) = P(A \cap R) + P(B \cap R)$

$P(R) = \frac{3}{12} \cdot \frac{6}{14} + \frac{9}{12} \cdot \frac{8}{18} = \frac{37}{84} = \boxed{0'44}$

a.2)

$P(A/R) = \frac{P(A \cap R)}{P(R)} = \frac{3/12 \cdot 6/14}{0'44} = \frac{9}{37} = \boxed{0'24}$

b)

$P(A) = 0'25$

$n = 6$

b.1.)

$P(X=1) = \binom{n}{r} p^r \cdot q^{n-r} = \binom{6}{1} \cdot 0'25^1 \cdot 0'75^5 \Rightarrow \text{TABLA} \Rightarrow \boxed{0'3560}$

b.2.)

$P(X \leq 5) = 1 - P(X=6) = 1 - \left[\binom{6}{6} 0'25^6 \cdot 0'75^0\right] \Rightarrow \text{TABLA} \Rightarrow 1 - 0'0002 =$

$= \boxed{0'9998}$

4.

$\pi \equiv ax + y - z = 1$

$A = (1,0,0)$

$B = (b,1,-1)$

a) $\vec{AB} \perp \pi$

$A \in \pi$

$\vec{AB} = (b-1, 1, -1)$

Para que $A \in \pi$,

$a \cdot 1 + 0 - 0 = 1$

$1a = 1$

$\boxed{a = 1}$

Para que $\vec{AB} \perp \pi$; $\vec{AB} \parallel \vec{n}$

$\vec{n} = (a, 1, -1)$

$\vec{AB} = \vec{n}$

$(b-1, 1, -1) = (a, 1, -1)$

$b - 1 = a$

$b - 1 = 1$

$\boxed{b = 2}$

b) $\pi \equiv x + y - z = 1$

$A = (1,0,0)$

$B = (2,1,-1)$

$r \begin{cases} \to A \in r \\ \to r \perp \pi \end{cases}$

El vector normal del plano es el vector director de la recta.

$\vec{n} = \vec{dr} = (1,1,-1)$

$$\boxed{r = \begin{cases} x = 1 + \lambda \\ y = \lambda \\ z = -\lambda \end{cases}}$$

5.

a) $x = -1$

$f(x) = x^2 - 2x + 3$

$g(x) = \frac{1}{2}x^2 + 1$

P. corte:

$x^2 - 2x + 3 = \frac{1}{2}x^2 + 1$

$+\frac{1}{2}x^2 - 2x + 2 = 0$

$\boxed{x = 2}$

$$A = \int_{-1}^{2} f(x) - g(x) = \int_{-1}^{2} x^2 - 2x + 3 - \frac{1}{2}x^2 - 1 = (*)$$

$$\left.\begin{matrix} f(0) = 3 \\ g(0) = 1 \end{matrix}\right\} f(0) > g(0)$$

$$(*) \int_{-1}^{2} \frac{1}{2}x^2 - 2x + 2 = \left(\frac{x^3}{6} - \frac{2x^2}{2} + 2x \right)_{-1}^{2} =$$

$$= \left(\frac{x^3}{6} - x^2 + 2x \right)_{-1}^{2} = \left(\frac{8}{6} - 4 + 4 \right) - \left(\frac{-1}{6} - 1 - 2 \right) =$$

$$= \frac{4}{3} - \left(\frac{-19}{6} \right) = \boxed{\frac{9}{2} u^2}$$

b)

$$A = \begin{pmatrix} -2 & 1 & a \\ -1 & 0 & 0 \\ -1 & a+1 & a+1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} -2 & 1 & a \\ -1 & 0 & 0 \\ -1 & a+1 & a+1 \end{vmatrix} = -a(a+1) + (a+1) = -a^2 - \cancel{a} + \cancel{a} + 1 = -a^2 + 1$$

$-a^2 + 1 = 0$

$a = \sqrt{1} = \boxed{\pm 1}$

· Para $a \neq \{-1, +1\} \rightarrow Rg\,A = 3$

· Para $a = \{-1, +1\} \rightarrow Rg\,A \leq 2$

$\boxed{a = 1}$

$|A| = \begin{vmatrix} -2 & 1 \\ -1 & 0 \end{vmatrix} = 1 \neq 0 \longrightarrow Rg\,A = 2$

$\boxed{a = -1}$

$|A| = \begin{vmatrix} -2 & 1 \\ -1 & 0 \end{vmatrix} = 1 \neq 0 \longrightarrow Rg\,A = 2$

6.

a) $\lim_{x \to 3} \frac{x^3 - 3x^2 + 3x - 9}{3x - 9} = \frac{0}{0}\ \text{IND} \overset{\text{L'H}}{=} \lim_{x \to 3} \frac{3x^2 - 6x + 3}{3} = \frac{12}{3} = \boxed{4}$

b)

$A = (1, 2, 1)$

$\pi \equiv x - y = 1$

$\vec{n}\,(1, -1, 0)$

$$d(A, \pi) = \frac{|A \cdot x + B \cdot y + C \cdot z + D|}{\sqrt{A^2 + B^2 + C^2}} =$$

$$= \frac{|1 \cdot 1 + (-1) \cdot 2 + 0 \cdot 1 + (-1)|}{\sqrt{1^2 + (-1)^2 + 0^2}} = \boxed{\frac{2}{\sqrt{2}}\ u}$$

7.

a) $\int (x+3)\cdot e^{-2x}\,dx = (x+3)\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{1}{2}e^{-2x}\,dx = \circledast$

$u = x+3 \qquad dv = e^{-2x}dx$

$du = 1dx \qquad V = -\frac{1}{2}e^{-2x}$

$\int e^{-2x}dx = \int e^t \frac{dt}{-2} = -\frac{1}{2}\int e^t dt = -\frac{1}{2}e^t + c = -\frac{1}{2}e^{-2x} + c$

$-2x = t$

$-2dx = dt$

$\circledast = (x-3)\left(-\frac{1}{2}e^{-2x}\right) + \frac{1}{2}\int e^{-2x}dx = (x-3)\left(-\frac{1}{2}e^{-2x}\right) + \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\right) + c =$

$= \boxed{\left(-\frac{1}{2}e^{-2x}\right)\left((x-3) + \frac{1}{2}\right) + c}$

b) $P(1) = \frac{1}{6}$

b.1.) $P(X=0) = \boxed{\frac{5}{6}} \qquad P(X=1) = \boxed{\frac{1}{6}\cdot\frac{5}{6}} \qquad P(X=3) = \left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) = \boxed{0{,}0038}$

b.2.) $P(X=n) = \left(\frac{1}{6}\right)^n\cdot\frac{5}{6}$

8.

a) $\begin{vmatrix} 1 & 3 & 5 \\ a & b & c \\ x & y & z \end{vmatrix} = 6$

$$\begin{vmatrix} 1/2 & 3/2 & 5/2 \\ a+2 & b+6 & c+10 \\ 4x & 4y & 4z \end{vmatrix} = \frac{1}{2}\cdot 4\begin{vmatrix} 1 & 3 & 5 \\ a+2 & b+6 & c+10 \\ x & y & z \end{vmatrix} = 2\left(\begin{vmatrix} 1 & 3 & 5 \\ a & b & c \\ x & y & z \end{vmatrix} + \overset{0}{\cancel{\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ x & y & z \end{vmatrix}}}\right) =$$

$= 2\cdot 6 = \boxed{12}$

b) $\vec{u} = (1,1,1)$

$\vec{v} = (3,2,3)$

$\cos\alpha = \frac{|\vec{u}\cdot\vec{v}|}{|\vec{u}|\cdot|\vec{v}|} = \frac{8}{\sqrt{3}\cdot\sqrt{22}} = 0'98$

$\vec{u}\cdot\vec{v} = 3+2+3 = 8 \qquad \alpha = \arccos 0'98 = \boxed{11{,}47^\circ}$

$|\vec{u}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$|\vec{v}| = \sqrt{3^2+2^2+3^2} = \sqrt{22}$