Junio 2024

① $$\overbrace{\left(\underbrace{\begin{array}{ccc} a & 2 & 1 \\ 2 & a & 1 \\ 5 & 2 & 1 \end{array}}_{A}\left|\begin{array}{c} 1 \\ a \\ 1 \end{array}\right.\right)}^{M^*}$$

∘ "Citar el Teorema Rouche - Frobenius"

a) $|A| = a^2-7a+10 = (a-5)(a-2)$

- $\boxed{\text{Si } a \neq 5, 2} \rightarrow |A| \neq 0 \rightarrow Rg(A) = Rg(M^*) = 3 \rightarrow \boxed{\text{S.C.D, 1 solución}}$
- $\boxed{\text{Si } a = 2}$

$$|A|_{2x2} = \begin{vmatrix} 2 & 2 \\ 5 & 2 \end{vmatrix} \neq 0 \rightarrow Rg(A) = 2$$

$$\underset{C_4 \rightarrow C_2}{|M^*|_{3x3}} = \begin{vmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix} = -3 \neq 0 \rightarrow Rg(M^*) = 3 \neq Rg(A)$$

$$\boxed{\text{S.I. , 0 soluciones}}$$

- $\boxed{\text{Si } a = 5}$

$$|A|_{2x2} = \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} \neq 0 \rightarrow Rg(A) = 2$$

$$\underset{C_4 \rightarrow C_1}{|M^*|_{3x3}} = \begin{vmatrix} 1 & 2 & 1 \\ 5 & 5 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 0 \qquad \underset{C_4 \rightarrow C_2}{|M^*|_{3x3}} = \begin{vmatrix} 5 & 1 & 1 \\ 2 & 5 & 1 \\ 5 & 1 & 1 \end{vmatrix} = 0$$

$$\underset{C_4 \rightarrow C_3}{|M^*|_{3x3}} = \begin{vmatrix} 5 & 2 & 1 \\ 2 & 5 & 5 \\ 5 & 2 & 1 \end{vmatrix} = 0 \longrightarrow Rg(M^*) = 2 = Rg(A) < 3$$

$$\boxed{\text{S.C.I , } \infty \text{ soluciones}}$$

b) Para $a=1$ el sistema es C.D, por lo que aplicamos la Regla de Cramer:

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 5 & 2 & 1 \end{vmatrix} = 4 \qquad \boxed{\text{Solución } (0,0,1)}$$

$$\left[X = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{|A|} = \frac{0}{4} = 0\right] \left[Y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 5 & 1 & 1 \end{vmatrix}}{|A|} = \frac{0}{4} = 0\right] \left[Z = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 5 & 2 & 1 \end{vmatrix}}{|A|} = \frac{4}{4} = 1\right.$$

②

$V = 1\,dm^3$

a) $A_T = 2A_B + 4A_L$

$A_B = x^2 \quad A_L = xy$

$A_T = 2x^2 + 4xy \to A_T(x) = 2x^2 + 4x \cdot \frac{1}{x^2}$

$\to \left[V = 1\,dm^3 = x^2y\right] \to y = 1/x^2$

$\boxed{A_T(x) = 2x^2 + \frac{4}{x}}$

b) Si A_T es mínimo $\to \begin{cases} A'_T = 0 \\ A''_T > 0 \end{cases}$ $\quad A'_T = 4x - \frac{4}{x^2}$

$A'_T = 0 = 4x - \frac{4}{x^2} \to x^3 = 1 \to \boxed{x = 1\,dm} \to \boxed{y = 1\,dm}$

$A''_T = 4 + \frac{8}{x^3} \to A''_T(1) = 4 + 8 > 0$ → es un mínimo

c) $\boxed{A_T(x=1) = 2 + 4 = 6\,dm^2}$

$\boxed{C = A_T \cdot \frac{5€}{dm^2} = 6 \cdot 5 = 30€}$

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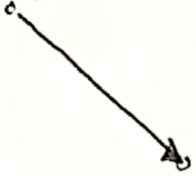
③ A(2,-1,3)



B(-2,4,5)

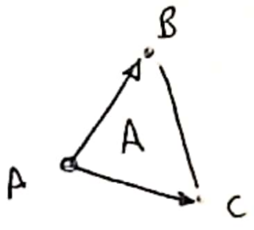
a) $r \equiv \begin{cases} \bar{V}_r = \overline{AB} \\ P_r = A = (2,-1,3) \end{cases}$

$\overline{AB} = B - A = (-4, 5, 2)$

$\boxed{r \equiv \frac{x-2}{-4} = \frac{y+1}{5} = \frac{z-3}{2}}$: ec. continua

b) $\boxed{L = |\overline{AB}| = \sqrt{(-4)^2 + 5^2 + 2^2} = \sqrt{45} = 3\sqrt{5}\ u}$

c)



C(0,0,1)

$\overline{AC} = C - A = (-2, 1, -2)$

$\overline{AB} \times \overline{AC} = \begin{vmatrix} i & j & k \\ -4 & 5 & 2 \\ -2 & 1 & -2 \end{vmatrix} = (-12, -12, 6)$

$\boxed{A = \frac{1}{2}|\overline{AB} \times \overline{AC}| = \frac{1}{2}\sqrt{(-12)^2 + (-12)^2 + 6^2} = 9u^2}$

④

a) $$\lim_{x\to\infty} \frac{e^x-1}{x^2+3} = \left(\frac{\infty}{\infty}\right)^{IND} \overset{*}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{*}{=} \lim_{x\to\infty} \frac{e^x}{2} = \boxed{+\infty}$$

* Aplicamos la regla de L'Hôpital

b) $$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 2 & 0 & 2 & 1 \\ a & 1 & 0 & 1 \end{pmatrix} \longrightarrow |A|_{2x2} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \neq 0 \rightarrow Rg(A) \geq 2$$

$$|A|_{3x3} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \qquad |A|_{3x3} = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ a & 1 & 0 \end{vmatrix} = 0$$

$$|A|_{3x3} = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 2 & 1 \\ a & 0 & 1 \end{vmatrix} = a \qquad |A|_{3x3} = \underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ a & 1 & 1 \\ C_1 & C_2 & C_4 \end{vmatrix}} = -1 \neq 0$$

- Debido a que C_1, C_2, C_4 son independientes para cualquier valor de a → $\boxed{Rg(A) = 3 \quad \forall a \in \mathbb{R}}$

⑤

a) $$\int x\sqrt{2x+3}\,dx = \int \left(\frac{t^2-3}{2}\right)\cdot t \cdot t\,dt = \int \frac{t^4}{2} - \frac{3}{2}t^2\,dt =$$

$$\left[\begin{array}{l} t = \sqrt{2x+3} \\ \frac{t^2-3}{2} = x \\ t\,dt = dx \end{array}\right] \quad = \frac{1}{2}\int t^4\,dt - \frac{3}{2}\int t^2\,dt = \frac{t^5}{10} - \frac{t^3}{2} =$$

$$= \boxed{\frac{(2x+3)^{5/2}}{10} - \frac{(2x+3)^{3/2}}{2} + C}$$

b) $\bar{u} = (1, a, a)$, $\bar{v} = (-1, 0, 2)$

$$\cos\alpha = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|} \rightarrow \cos 60^\circ = \frac{1}{2} = \frac{1\cdot(-1) + a\cdot 0 + 2\cdot a}{\sqrt{1^2+a^2+a^2}\cdot\sqrt{(-1)^2+2^2}}$$

$$\frac{1}{2} = \frac{2a-1}{\sqrt{1+2a^2}\sqrt{5}} \; ; \quad \sqrt{5+10a^2} = 4a-2$$

$$; \; 5+10a^2 = (4a-2)^2 = 16a^2 ; 16a + 4$$

$$6a^2 - 16a - 1 = 0 \longrightarrow a = \frac{16 \pm \sqrt{16^2-4\cdot 6\cdot(-1)}}{2\cdot 6} = \frac{16 \pm 2\sqrt{70}}{12}$$

Deseamos quedarnos con el valor positivo para que $\cos 60 > 0 \leftrightarrow \vec{u}\cdot\vec{v} > 0$

$$\boxed{a = \frac{8+\sqrt{70}}{6}}$$

⑥

a) $f(x) = x^3 + ax^2 + bx + c$

- Extremo relativo en $x = 2 \rightarrow f'(2) = 0$
- Punto inflexión en $x = 1 \rightarrow f''(1) = 0$
- $P(1,2) \longrightarrow f(1) = 2$
- $f(1) = 2 = 1^3 + a\cdot 1^2 + b\cdot 1 + c \longrightarrow \boxed{a+b+c+1 = 2}$
- $f'(x) = 3x^2 + 2ax + b \rightarrow f'(2) = 0 \rightarrow \boxed{4a + b + 12 = 0}$
- $f''(x) = 6x + 2a \rightarrow f''(1) = 0 \rightarrow \boxed{2a + 6 = 0} \rightarrow \boxed{a = -3}$

Sustituyendo $a = -3 \rightarrow \boxed{b = 0} \rightarrow \boxed{c = 4}$

b1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\boxed{P(B) = P(A\cup B) - P(A) + P(A \cap B) = 0'3 - 0'2 + 0'1 = 0'2}$

- $\boxed{P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0'2 - 0'1 = 0'1}$

b2) $\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0'1}{0'2} = 0'5}$ $\boxed{P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{0'1}{0'2} = 0'5}$

* Teorema de Bayes

⑦

a) $A = \begin{pmatrix} a & 1 \\ 1 & 0 \end{pmatrix} \rightarrow$ Si $A = A^{-1} \rightarrow \left[A\cdot A^{-1} = A\cdot A = I\right]$

$$\begin{pmatrix} a & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} a^2+1 & a \\ a & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Solo se cumple para $\boxed{a = 0}$

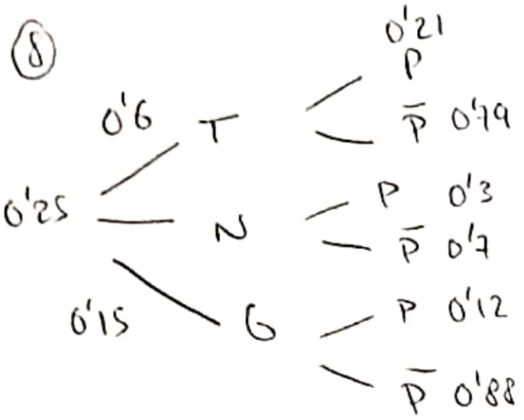
Escaneado con CamScanner

7 b)

$$r \equiv \begin{cases} A(1,0,0) \\ \perp \ \bar{u}, \bar{v} \longrightarrow \bar{v}_r = \bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{vmatrix} = (0,1,2) \end{cases}$$

$$\left[r \equiv \begin{cases} x = 1 \\ y = t \\ z = -2t \end{cases} \quad \text{Ec. paramétrica} \right]$$

⑧



a1) $P(P) = P(T \cap P) + P(N \cap P) + P(G \cap P)$

$P(P) = 0'6 \cdot 0'21 + 0'25 \cdot 0'3 + 0'15 \cdot 0'12$

$\boxed{P(P) = 0'219}$

a2) $\left[P(N|P) = \frac{P(N \cap P)}{P(P)} = \frac{0'25 \cdot 0'3}{0'219} \simeq 0'34 \right]$

b1) $X \sim N(60,8)$ $\quad z = \frac{x - \mu}{\sigma}$ $\quad z = \frac{50-60}{8} = -1'25$

$\boxed{P(X \leq 50)} = P(z \leq -1'25) = 1 - P(z \leq 1'25) = 1 - 0'8944 = \boxed{0'1056}$

b2) $\boxed{P(50 \leq X < 66)} = P(-1'25 \leq z < 0'75) = P(z < 0'75) - P(z < -1'25) =$

$z = \frac{66-60}{8} = 0'75$ $\quad = 0'7733 - 0'1056 = \boxed{0'6678}$